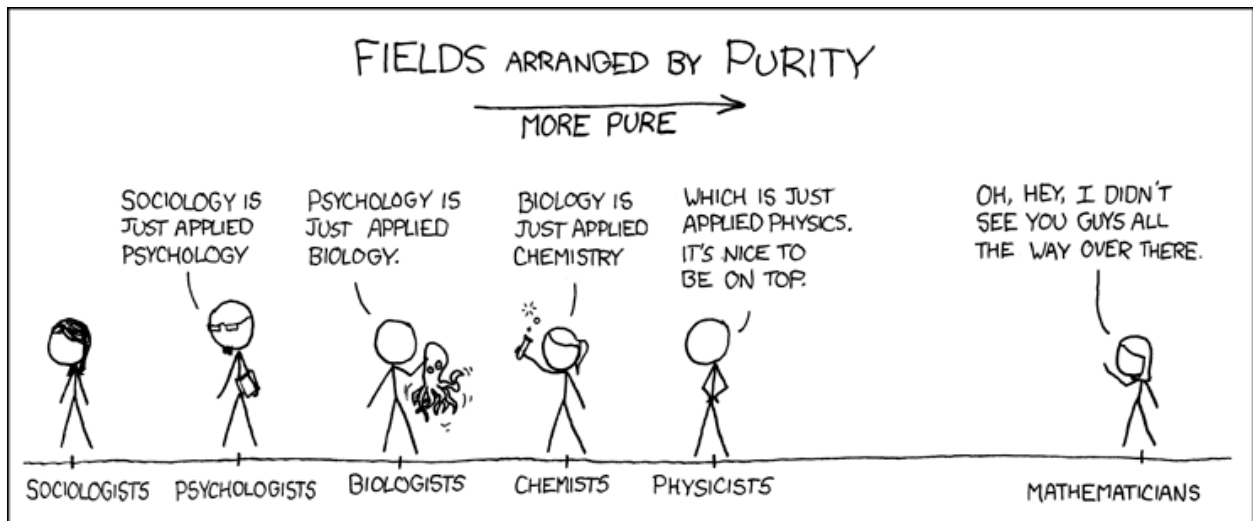# EXAM 2

- The exam is closed book, notes and neighbor. No calculators.
- SHOW ALL WORK!!!
- Good luck!

Problem	1	2	3	4	5	6	Bonus	Total
Score								
Possible	14	16	16	20	16	18	10	100

1. (14 points) Consider the differential equation below.

$$x''' + 6x'' + 9x' = g(x)$$

    a. (6 points) Find the complementary solution to the homogeneous differential equation.

    b. (8 points) For the DE described above, let $g(x)$ be defined below. For each different $g(x)$, guess the form of the particular solution and give the proper annihilator function. Use the space below for work before writing your answer.

i. $g(x) = 4x^3$ Guess form of y_p _____

                                  Annihilator Function   _____

ii. $g(x) = 5xe^{-3x}$ Guess form of y_p _____

                                  Annihilator Function   _____

iii. $g(x) = xe^{-3x}\sin(4x)$ Guess form of y_p _____

                                  Annihilator Function   _____

Space below is for work if needed.

2. (16 points) Solve the differential equation using the **superposition approach** for undetermined coefficients. Little or no credit will be given if the annihilator approached is used.

$$y'' - y' - 2y = 6x + 6e^{-x}$$

3. (16 points) Solve the system of differential equations.

$$\begin{cases} x' = x - 2y \\ y' = 5x - y \end{cases}$$

4. (20 points) A spring attached to the ceiling is stretched by 6 inches by a 2lb weight. The spring is in a medium that imparts a damping force of 1lb·sec/ft. The mass is pulled down 3 inches and imparted with an upward velocity of 3 ft/s.

   a. (7 points) What is the DE of the system and the initial conditions?

   b. (11 points) Find the equation of motion that satisfies the initial conditions.

5. (16 points) Use **variation of parameters** to solve the non-homogeneous differential equation.

$$x^2 y'' + 8xy' + 6y = x^{-1}$$

6. (18 points) Use the **reduction of order method** to find the second and particular solutions of the DE.

$$x^2y'' + 8xy' + 6y = x^2 \qquad\qquad y_1(x) = x^{-1}$$

Bonus:

a. (4 points) What is the particular solution for the differential equation below? (Hint: this DE is related to problems 5 and 6). **Explain** how you arrived at your answer.

$$x^2 y'' + 8xy' + 6y = x^2 + x^{-1}$$

b. (6 points) State the intervals where solutions may exist for the differential equation. State the longest interval on which the DE is certain to have a unique solution for the initial conditions given.

$$(t-1)y'' - 3ty' + 4y = \sin(t) \qquad y(-2) = 2, \quad y'(-2) = 1$$